

Disproof of Bell's Theorem

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We illustrate an explicit counterexample to Bell's theorem by constructing a pair of dichotomic variables that exactly reproduce the EPR-Bohm correlations in a manifestly local-realistic manner.

Central to Bell's theorem [1] is the claim that no local and realistic model can reproduce the correlations observed in the EPR-Bohm experiments. Here we construct such a model. Let Alice and Bob be equipped with the variables

$$A(\mathbf{a}, \lambda) = \{-a_j \beta_j\} \{a_k \beta_k(\lambda)\} = \begin{cases} +1 & \text{if } \lambda = +1 \\ -1 & \text{if } \lambda = -1 \end{cases} \quad (1)$$

$$\text{and } B(\mathbf{b}, \lambda) = \{b_j \beta_j(\lambda)\} \{b_k \beta_k\} = \begin{cases} -1 & \text{if } \lambda = +1 \\ +1 & \text{if } \lambda = -1, \end{cases} \quad (2)$$

where the repeated indices are summed over x, y , and z ; the fixed bivector basis $\{\beta_x, \beta_y, \beta_z\}$ is defined by the algebra

$$\beta_j \beta_k = -\delta_{jk} - \epsilon_{jkl} \beta_l; \quad (3)$$

and—together with $\beta_j(\lambda) = \lambda \beta_j$ —the λ -dependent bivector basis $\{\beta_x(\lambda), \beta_y(\lambda), \beta_z(\lambda)\}$ is defined by the algebra

$$\beta_j \beta_k = -\delta_{jk} - \lambda \epsilon_{jkl} \beta_l, \quad \text{where } \lambda = \pm 1 \text{ is a fair coin [2],} \quad (4)$$

δ_{jk} is the Kronecker delta, ϵ_{jkl} is the Levi-Civita symbol, $\mathbf{a} = a_x \mathbf{e}_x + a_y \mathbf{e}_y + a_z \mathbf{e}_z$ and $\mathbf{b} = b_x \mathbf{e}_x + b_y \mathbf{e}_y + b_z \mathbf{e}_z$ are unit vectors, and the indices $j, k, l = x, y$, or z . The correlation between $A(\mathbf{a}, \lambda)$ and $B(\mathbf{b}, \lambda)$ then works out to be

$$\mathcal{E}(\mathbf{a}, \mathbf{b}) = \frac{\lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \sum_{i=1}^n A(\mathbf{a}, \lambda^i) B(\mathbf{b}, \lambda^i) \right\}}{\{-a_j \beta_j\} \{b_k \beta_k\}} = \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n \frac{A(\mathbf{a}, \lambda^i) B(\mathbf{b}, \lambda^i)}{\{-a_j \beta_j\} \{b_k \beta_k\}} \right] \quad (5)$$

$$= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n \{a_j \beta_j\} \{A(\mathbf{a}, \lambda^i) B(\mathbf{b}, \lambda^i)\} \{-b_k \beta_k\} \right] = \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n \{a_j \beta_j(\lambda^i)\} \{b_k \beta_k(\lambda^i)\} \right] \quad (6)$$

$$= -a_j b_j - \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{i=1}^n \{ \lambda^i \epsilon_{jkl} a_j b_k \beta_l \} \right] = -a_j b_j + 0 = -\mathbf{a} \cdot \mathbf{b}, \quad (7)$$

where the denominators in (5) are standard deviations. The corresponding CHSH string of expectation values gives

$$|\mathcal{E}(\mathbf{a}, \mathbf{b}) + \mathcal{E}(\mathbf{a}, \mathbf{b}') + \mathcal{E}(\mathbf{a}', \mathbf{b}) - \mathcal{E}(\mathbf{a}', \mathbf{b}')| \leq 2 \sqrt{1 - (\mathbf{a} \times \mathbf{a}') \cdot (\mathbf{b}' \times \mathbf{b})} \leq 2\sqrt{2}. \quad (8)$$

Evidently, the variables $A(\mathbf{a}, \lambda)$ and $B(\mathbf{b}, \lambda)$ defined above respect both the remote parameter independence and the remote outcome independence (which has been checked rigorously [2][3][4][5][6][7]). This contradicts Bell's theorem.

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References

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